Instructions: This is a written project and you are expected to write down detailed explanations and arguments to justify your calculations. If you wish to use a certain formula, be sure to state it clearly and give a brief justification for it.

Just a numerical answer, even when correct, will receive little credit.

Preamble: You will need the concept of \( \int_C P \, dx + Q \, dy \) where \( P, Q \) are functions of \( x, y \) and \( C \) is a piecewise parametrized closed curve.

The problem: For a curve \( C \) parametrized by a parametrization \( <x, y> = <u(t), v(t)> \) on an interval \([a, b]\), we define the integral

\[
\int_C P \, dx + Q \, dy = \int_{t=a}^{b} \left( P(u(t), v(t))u'(t) + Q(u(t), v(t))v'(t) \right) \, dt.
\]

If a closed curve is formed by successive pieces \( C_1, C_2, \cdots, C_r \), then we define integral over \( C \) as the sum of integrals over all the pieces.

You are asked to compute several such curve integrals. and make appropriate observations.

(1) (6 points) Let \( C \) be the triangle \( \triangle LMN \) where \( L = (0, 0), M = (2, 0) \) and \( N = (1, 1) \), traced counterclockwise.
   (a) (2 points) Parameterize the line segments \( LM, MN, \) and \( NL \)

   Answer:

   (b) (4 points) Given that the integral is additive, i.e.:

   \[
   \int_C x \, dy = \int_{LM} x \, dy + \int_{MN} x \, dy + \int_{NL} x \, dy
   \]

   Calculate each piece of the integral separately, and add the result.

   Answer:

(2) (3 points) Let \( C \) be the circle of radius 2 with center \((0, 0)\) traced counterclockwise. Parameterize the circle \( C \) and calculate the integral \( \int_C x \, dy \).

Answer:

(3) (1 point) Use basic geometry to calculate the area of \( \triangle LMN \) and the area of the circle of radius 2.

Answer:

(4) Extra Credit

Prove that changing \( N \) to \((k, 1)\) for an arbitrary real number \( k \), in problem 1 above, yields the same conclusion.

What theorem does this suggest to you? Why?

Answer: