Old Exam 2 problems solved
Ma 110 Fall 2008

We give short explanations of old exam problems related to the current exam 2 syllabus. You need to look up the original problems from the on line copies of the old exams on the course web site.

Old Ex 2 relevant problems.

Q.1. The relation between the two coordinate systems on the line is $x' = ax + b$. Use information about the points A, B to get the following two equations:

$$0 = a(3) + b \text{ and } 1 = a(\frac{5}{2}) + b.$$

Subtracting second from first, we get: $-1 = a(\frac{1}{2})$ or $a = -2$. Plugging this into either of the two equations, get $b = 6$.

Thus the general formula is: $x' = -2x + 6$.

Now use the formula to find the x' coordinates of O and P respectively,

$$-2(0) + 6 = 6 \text{ and } -2(1) + 6 = 4.$$

The x-coordinate of R is found by solving $5 = -2(x) + 6$ and it is $\frac{1}{2}$.

Q.2. Assume the multiplier number is $C = a+ib$ and thus $T(P)$ as a complex number is given by

$$(a + ib)(x + iy) = ax - by + i(ay + bx).$$

Using the given point A and its image $T(A)$, we get:

$$14 - 5i = a(2) - b(3) + i(a(3) + b(2)) \text{ or } 2a - 3b = 14, 3a + 2b = -5.$$

Use any convenient method to solve these and get $a = 1, b = -4$. Thus $C = 1 - 4i$.

Thus the transformation T sends (x, y) to $(x + 4y, y - 4x)$.

The point $T(B)$ is found by using $B(-7, 1)$ to get

$$T(B) = (-7 + 4, 1 - 4(-7)) = (-3, 29).$$

Q.3. As usual, assume that the relation between the two coordinate systems (x for the A-people and x' for the B-people) is $x' = ax + b$.

Note the fact that the A-town itself would have coordinates $x = 0$ and $x' = 100$. Thus, we have $100 = a(0) + b$ or $b = 100$.

Similarly, for the B-town, we have $x' = 0$ and $x = 160$. So, $0 = a(160) + 100$ where **we have used the known value** of b. Thus $a = -\frac{100}{160} = -\frac{5}{8}$ and the formula is:

$$x' = -\frac{5}{8}x + 100.$$

Now the given border is 30 miles from A, i.e. $70 = 100 - 30$ miles from B. Thus the border has $x' = 70$. The question (a) was actually asking only this much!

We do more. Then x-coordinate for the border (distance in km from A) is found by solving:

$$70 = -\frac{5}{8}x + 100 \text{ so } \frac{5}{8}x = 30.$$

This gives $x = 48$. So the border is 48km from A.

Old Ex 3 relevant problems.

Q.1. The (natural) parametric equations of the line joining A, B are given by

$$(x, y) = (1 - t)A + tB = (1 - t)(3, 7) + t(5, 11).$$

This simplifies to:

$$x = (1 - t)(3) + t(5) = 2t + 3 \text{ and } y = (1 - t)(7) + t(11) = 4t + 7.$$

This parameterization corresponds to $A(0), B(1)$ as parameter values.

Note that the question does not specifically state this as a requirement, so many alternate answers are possible. This is the simplest answer to give!

Q.2. Take the two equations $x = 2t + 1, y = 4t - 3$ and eliminate t by calculating:

$$2x - y = 2(2t + 1) - (4t - 3) = 2 + 3 = 5.$$

Thus the desired answer is $y = 2x - 5$.

Q.3. The direction numbers for the line joining A, B are $\overrightarrow{AB} = B - A = (2, 4)$. So the perpendicular line would have direction numbers $(4, -2)$.

The midpoint of the segment AB is: $\frac{A+B}{2} = (\frac{3+5}{2}, \frac{7+11}{2}) = (4, 9)$. **Note corrected number here.**

The required perpendicular bisector is then a line passing through $M(4, 9)$ with direction numbers $(4, -2)$.

So the parametric form of the perpendicular bisector is:

$$(x, y) = (4, 9) + t(4, -2) \text{ or } x = 4 + 4t, y = 9 - 2t.$$

Q.4. The part (ii) in (a) is misplaced and some lines are duplicated.

Here are the correct questions and answers.

(a) If the shown line has equation $y = mx + b$, then from the picture, we see that the slope is positive, so $m > 0$ and the y-intercept b is negative.
Thus, the sign of $-b/m$ is positive. This answers (i),(iii).

(b) If the shown parabola is the graph of $y = ax^2 + bx + c$, then clearly $a < 0$ because it opens below.
The vertex has x-coordinate negative, so $-b/(2a) < 0$.
Thus, we must have $b < 0$ (since $a, -b/(2a)$ are negative.)
The y-intercept is c and hence from picture $c > 0$.
There are two x-intercepts and these are obtained by solving $ax^2 + bx + c = 0$.
Thus, the quadratic formula must give two real answers and hence its discriminant $b^2 - 4ac$ must be positive!
You could also deduce this from the known fact that both its terms are now shown to be positive!
If the roots of the quadratic are r_1, r_2, then we know the identity:
We know that the x-coordinate of the vertex $-b/(2a)$ is equal to the average $\frac{r_1+r_2}{2}$ of the two roots. Thus we know this average is negative too!
The largest value of $ax^2 + bx + c$ is the y-coordinate of the vertex which is visibly positive!

Q.5. The slope of the line joining $(1, -2)$ and $(6, t)$ is $\frac{t+2}{5}$. The perpendicularity condition says:

$$\frac{t+2}{5} \cdot (5) = -1 \text{ or } t + 2 = -1.$$

So $t = -3$.

Q.7. Given two parametric lines, find the point of intersection.

We recommend that you convert one of them to a usual form. Thus, line 1 is $x = t + 3, y = -2t + 3$. For line 2 $(x = -s + 2, y = 3s + 4)$ we see $y + 3x = 4 + 6 = 10$.

To intersect line 1 with line 2 we substitute the line 1 parametric equations into the equation for line 2.

$$(-2t + 3) + 3(t + 3) = 10 \text{ or } t + 3 + 9 = 10 \text{ or } t = -2.$$

To find the actual point, substitute this in the parametric form of line 1: $(x, y) = (-2 + 3, -2(-2) + 3) = (1, 7)$.

Q.8. The area of the extended triangle is $\frac{1}{2}(2+x)(3+x)$ and we equate this to 45. Thus the equation is:

$$(2 + x)(3 + x) = 90 \text{ or } x^2 + 5x - 84 = 0.$$

Solve this to deduce $x = 7$ or $x = -12$.

Clearly $x = -12$ is unacceptable for the geometric construction, so $x = 7$ is the answer.

Q.9. Just plug in the points $(-1, 9)$ and $(1, 3)$ into the proposed equation $y = x^2 + bx + c$ and solve for b, c.

$$9 = 1 - b + c, 3 = 1 + b + c.$$

Subtracting one from the other, we get $6 = -2b$, so $b = -3$. From the first equation, we deduce $9 = 1 + 3 + c$ so $c = 5$. Thus the parabola is $y = x^2 - 3x + 5$.