

## Old Exam 1 problems solved

### Ma 110 Fall 2008

We give short explanations of old exam problems related to the current exam 1 syllabus. You need to look up the original problems from the on line copies of the old exams on the course web site.

Old Ex 1

Q.1.

$$\frac{17}{23} - \frac{31}{19} = \frac{(17)(19) - (31)(23)}{(23)(19)} = \frac{-390}{437}.$$

Q.2. Given:

$$\frac{X+1}{X-2} = \frac{4}{5} = \frac{A}{B}.$$

Thus  $5(X+1) = 4(X-2)$  or  $X = -8 - 5 = -13$ .

Q.3. **not in current syllabus at this point.** The ratio of the circumference to the arc is  $\frac{2\pi(10)}{10} = 2\pi$ . If the given angle is called  $t$  radians, then  $\frac{2\pi}{t}$  is also equal to  $2\pi$ , so  $t = 1$ .

The ratio of the areas will also be  $2\pi$ . So  $\frac{2\pi(10)^2}{A} = 2\pi$  where  $A$  is the desired area. Hence.  $A = 100$ .

Q.4a. Recall that  $\frac{1}{a+bi} = \frac{a-bi}{a^2+b^2}$ . So

$$\frac{1}{3-5i} = \frac{3+5i}{9+25} = \frac{3}{34} + \frac{5}{34}i.$$

Q.4b. If  $(3-5i)X = 11-7i$  then  $X = \frac{11-7i}{3-5i}$ .

Using the above result, we quickly get:

$$X = \frac{(11-7i)(3+5i)}{34} = \frac{68+34i}{34} = 2+i.$$

Q.5. The coefficient of  $x^4$  when  $(x^3 - 5x^2 + 7x - 1)(2x^2 + x - 1)$  is multiplied out comes from adding up the terms:

$$(x^3)(x) - (5x^2)(2x^2) = x^4 - 10x^4 = -9x^4.$$

All other terms give different powers of  $x$ .

Q.6a.  $(2A - 3B)^5$  is expanded. Then the coefficient of  $A^2B^3$  comes from the term

$${}_5C_3(2A)^2(-3B)^3 = \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} 2^2(-3)^3 A^2 B^3$$

hence the answer is

$$\frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} 2^2(-3)^3 = -10 \cdot 4 \cdot 27 = -1080.$$

Q.6b.  $(x-y)^{17}$  is expanded. Then the coefficient of  $x^{14}y^3$  comes from  ${}_{17}C_3x^{14}(-y)^3$  and is evaluated from

$$\frac{17 \cdot 16 \cdot 15}{1 \cdot 2 \cdot 3} (-1)^3 x^{14} y^3 = -680x^{14}y^3$$

, as  $-680$ .

Q.7. The equations are  $2x + 3y = 11, 4x + 2y = 6$ . So, by Cramer's Rule:

$$x = \frac{\begin{vmatrix} 11 & 3 \\ 6 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix}} = \frac{22 - 18}{4 - 12} = -\frac{1}{2}.$$

$$y = \frac{\begin{vmatrix} 2 & 11 \\ 4 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix}} = \frac{12 - 44}{4 - 12} = 4.$$

Q.8. Let the numbers be  $x, y$  so that we have  $x + y = 7$  and  $2x + 3y = 9$ . Any convenient method gives  $x = 12$  and  $y = -5$ .

1. (a) The determinant  $\begin{vmatrix} 7 & -3 \\ 4 & 5 \end{vmatrix} = 35 - (-12) = 47$ .

(b) The given polynomials are such that  $f(x)$  has degree 5 and  $g(x)$  has degree 3. Then by the product rule,  $f(x)g(x)$  has degree  $5 + 3 = 8$ . The degree of  $x^2g(x)$  is  $2 + 3 = 5$ . Thus, for the degree of  $f(x) + x^2g(x)$  we need to actually check cancellation, since both terms have equal degree.

Calculation gives:

$$x^5 - 4x^3 + \text{lower terms} + x^2(-x^3 + x^2 + 1) = -4x^3 + \text{lower terms} + x^4 + x^2$$

so the resulting degree is 4.

Old Ex 2 relevant problems.

Q.4a. Definition of GCD is to be completed. Missing part is: “then  $q|d$ .”

Q.4b. A monic polynomial of degree three is given as  $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$ . Then  $a_4 = 0$ , since the degree is smaller than 4 and  $a_3$  is 1, since it is monic.

Q.4c. The missing part is “then  $L|q$ .”

Q.4d. Definition of the division algorithm. Since the divisor has degree 3, the remainder is zero or has degrees 0, 1, 2.

The quotient has degree equal to

“degree of dividend - degree of divisor ” or  $(7 + 5) - 3 = 9$ .

Q.5a. Given:

$$2567 = a_0 + a_15 + a_25^2 + a_35^3 + a_45^4 + a_55^5 + a_65^6 + a_75^7.$$

$a_0 = 2$  since it is the remainder of the right hand side divided by 5. Now move it to left and divide both sides by 5 to write

$$513 = a_1 + a_25 + \cdots a_75^6.$$

Now,  $a_1$  is the remainder of 513 modulo 5, so 3.

Thus, successive numbers can be calculated.

A useful observation is that  $a_5, a_6, a_7$  are all zero.

To see this, note that  $5^5$  and hence all bigger powers of 5 are bigger than 2567. So, all these higher coefficients will be zero.

Q.5b. The weekdays repeat after every 7 days. Thus from 596 we can throw away multiples of 7 leaving a remainder of 1. Thus the weekday is the same as one day after or Saturday.

Q.6a. Complete Aryabhata table.

$$\begin{pmatrix} \textit{Begin} & 1 & 0 & 268 \\ -1 & 0 & 1 & 244 \\ -10 & 1 & -1 & 24 \\ -6 & -10 & 11 & 4 \\ \textit{End} & 61 & -67 & 0 \end{pmatrix}$$

Q.6b. GCD is  $d = 4$ .

Q.6c.

$$d = (-10)268 + (11)244$$

Q.6d.

$$LCM = (268)(244)/4 = (61)(268) = (67)(244) = 16348.$$

Q.7. We have, from the Aryabhata Table, two expressions:

$$GCD = 3 = (-5)39 + (6)33 = (6)39 + (-7)33.$$

The second is obtained from the first by adding the last entries  $(11, -13)$  to the corresponding entries  $(-5, 6)$  above it.

Now multiplying by 5, we get our 15 gm weight expressed as:

$$15 = (-25)39 + (30)33 = (30)39 + (-35)33.$$

This gives two solutions.

Since  $15 + (25)39 = (30)33$  we can put the 15 gm weight together with 25 39 gm weights in the left pan and 30 33 gm weights in the right.

Similarly, we could use the second choice  $15 + (35)33 = (30)39$ .

If we wish to use the least number of weights, then the first solution is desired.

Q.7b. If we try to weigh a 2 gm weight, then 2 would be a combination of 39, 33. Since the GCD 3 does not divide 2, this is impossible.

Q.9. Assume that the walkway has  $x$  blue bricks (32 inches long) and  $y$  yellow bricks (22 inches long).

Then we must have the length  $L$  of the walkway as  $L = 32x = 22y$ . Thus  $L$  is divisible by both 32, 22, so the smallest value is the LCM. This comes out  $(32)(22)/2$  since the GCD is 2. The answer is  $L = 352$  inches. This is made by 11 blue and 16 yellow bricks.