Worksheet #3: Volumes of Rotation

1. Sketch the solid given by the integral

\[ \pi \int_0^1 (y^2 + 1)^2 - 1 \, dy. \]

2. Find the volume of the solid obtained by rotating the region bounded by \( y = \frac{1}{x^3}, \ y = 0, \ x = 1, \ \) and \( x = 6, \) about the \( x \)-axis.

3. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis. \( y = 0, \ y = \cos(2x), \ x = \frac{\pi}{2}, \ x = 0 \) about the line \( y = -6. \)

4. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by the curves \( x = 0, \ y = 1, \ x = y^{1/11}, \) about the line \( y = 1. \)

5. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).
   (a) \( R \) is region bounded by \( y = 1 - x^2 \) and \( y = 0; \) about the \( x \)-axis.
   (b) \( R \) is region bounded by \( y = \frac{1}{x}, \ x = 1, \ x = 2, \ \) and \( y = 0; \) about the \( x \)-axis.
   (c) \( R \) is region bounded by \( x = 2\sqrt{y}, \ x = 0, \) and \( y = 9; \) about the \( y \)-axis.
   (d) \( R \) is region bounded by \( y = 1 - x^2 \) and \( y = 0; \) about the line \( y = -1. \)
   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?
   (f) \( R \) is region bounded by \( y = e^{-x}, \ y = 1, \) and \( x = 2; \) about the line \( y = 2. \)
   (g) \( R \) is region bounded by \( y = x \) and \( y = \sqrt{x}; \) about the line \( x = 2. \)

6. Find the volume of the cone obtained by rotating the region under the segment joining \((0, h)\) and \((r, 0)\) about the \( y \)-axis.

7. A soda glass has the shape of the surface generated by revolving the graph of \( y = 6x^2 \) for \( 0 \leq x \leq 1 \) about the \( y \)-axis. Soda is extracted from the glass through a straw at the rate of \( 1/2 \) cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)

8. The torus is the solid obtained by rotating the circle \( (x - a)^2 + y^2 = b^2 \) around the \( y \)-axis (assume that \( a > b. \)) Show that it has volume \( 2\pi^2 ab^2. \)
   [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]